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**EQUIVALENT LINEARIZATION FOR FATIGUE
LIFE ESTIMATES OF A NONLINEAR
STRUCTURE**

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R. N. MILES

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EQUIVALENT LINEARIZATION FOR FATIGUE LIFE ESTIMATES OF A NONLINEAR STRUCTURE

R. N. Miles*
Department of Mechanical Engineering
University of California
Berkeley, CA 94720

Abstract

An analysis is presented of the suitability of the method of equivalent linearization for estimating the fatigue life of a nonlinear structure. Comparisons are performed of the fatigue life of a nonlinear plate as predicted using conventional equivalent linearization and three other more accurate methods. The excitation of the plate is assumed to be Gaussian white noise and the plate response is modeled using a single resonant mode. The methods used for comparison consist of numerical simulation, a probabilistic formulation, and a modification of equivalent linearization which avoids the usual assumption that the response process is Gaussian. Remarkably close agreement is obtained between all four methods, even for cases where the response is significantly nonlinear.

Introduction

Equivalent linearization has been applied to predict the mean square responses of a variety of complex nonlinear systems. However, the applicability of the method for the prediction of other response parameters such as the power spectral density and higher order statistics is severely limited by the assumption that the probability density of the system response is Gaussian. It is known that the response of a nonlinear system with Gaussian input will not be Gaussian. It has

* Present address:
Department of Mechanical and Industrial Engineering
State University of New York
Binghamton, NY 13901

been shown recently¹ that the response spectrum predicted by equivalent linearization for a Duffing oscillator differs substantially from that obtained by numerical simulation. Numerical simulations have shown that as the system is excited by higher level inputs, the resonant response peak, observed in the power spectrum, tends to broaden. Conventional equivalent linearization does not predict this effect. A modification to equivalent linearization has been developed, however, which avoids the assumption that the response is Gaussian and accurately predicts the broadened spectral response at high excitation levels.¹ Since conventional equivalent linearization is not suitable for predicting spectral response, there is a concern that it may not be useful for fatigue life estimation. In the study described herein, it is shown that, in spite of its shortcomings, conventional equivalent linearization provides fatigue life estimates which agree closely with those obtained by more sophisticated methods for a system which vibrates nonlinearly in a single resonant mode.

The estimation of fatigue life typically depends on an accurate description of the response probability density and the response spectrum. Both of these parameters influence the probability density of response peaks which is a key ingredient in fatigue estimation. Both of these factors are very crudely represented in conventional equivalent linearization. The method remains quite attractive, however, since a detailed analysis of the fatigue life of a complex nonlinear structure using more accurate methods such as numerical simulation or probabilistic techniques is not presently feasible unless the structure has a very limited number of degrees of freedom. If equivalent linearization could be applied directly to estimate the fatigue life of a complex nonlinear structure, it would avoid either the extensive numerical complexities of time domain simulations, or the mathematical intricacies of obtaining a detailed probabilistic description.

In the following, an overview will be presented of basic fatigue life estimation for a structure which experiences random loads. 'Classical' analytical methods will then be discussed which may be used to obtain fatigue life estimates for linear and nonlinear structures having a single degree of freedom. A numerical simulation approach is then presented. The application of conventional equivalent linearization for estimating fatigue life is then described. Finally, estimates of the fatigue life are presented using a modified equivalent linearization approach which avoids the Gaussian assumptions.¹ These techniques are then applied to predict the fatigue life of a nonlinear plate driven by Gaussian white noise. Remarkably good agreement is obtained between all methods. These results indicate that it may be possible to adapt equivalent linearization to predict the fatigue life of complex, multi-mode, nonlinear systems.

Fatigue Life Estimates for a System with Random Stresses

For the present study we will assume that the relation between the stress amplitude, S , and the number of cycles to failure N , may be approximated by the well known relation,

$$N = \frac{c}{S^b} \quad (1)$$

where c and b are experimentally obtained constants for a given material. It will be assumed that the number of fatigue cycles experienced by the structure is equal to the number of positive stress peaks, or stress reversals, that occur over time. A sample time history is shown in Figure 1 where the positive peaks are identified by circles.

The assumption that damage occurs at each positive stress peak may be conveniently combined with the Palmgren-Miner linear damage accumulation rule²

to produce fatigue life estimates. In this theory, the damage, D_i , caused by stress reversals at the stress level S_i is

$$D_i(S_i) = \frac{n(S_i)}{N(S_i)} \quad (2)$$

where $n(S_i)$ is the number of stress reversals experienced by the structure at the stress level S_i and $N(S_i)$ is the number of reversals required to cause failure at this stress level. The total damage, D_m , will be the sum of the damage at all stress levels that occur,

$$D_m = \sum_i D_i(S_i) = \sum_i \frac{n(S_i)}{N(S_i)} \quad (3)$$

From equation (1),

$$N(S_i) = \frac{c}{S_i^b} \quad (4)$$

so that equation (3) becomes

$$D_m = \frac{1}{c} \sum_i n(S_i) S_i^b \quad (5)$$

The form of equation (1) assumes that all stress peaks occur at positive stress levels as would be the case for sinusoidal loading. When a resonant system is subjected to random loading, however, we must allow for the possibility of stress peaks at negative stresses which make positive contributions to the accumulated damage. To account for this, we will take the absolute value of the stress level, S_i , so that equation (5) becomes

$$D_m = \frac{1}{c} \sum_i n(S_i) |S_i|^b \quad (6)$$

Failure is predicted to occur when $D_m=1$.

Since the response and rate of damage accumulation in the structure is assumed to be random, the fatigue life may be estimated from the expected value of

the rate of damage accumulation. This may be expressed as an integration over all possible stress levels in the form

$$E[D_m] = \int_{-\infty}^{\infty} n(S) \frac{|S|^b}{c} P(S) dS \quad , \quad (7)$$

where $P(S)$ is the probability density for response peaks.

The assumption that the response process is stationary gives the fatigue life as

$$T = \frac{1}{E[D_m]} \quad . \quad (8)$$

In the case where the system is linear and has a single lightly damped resonant mode, the peak probability density may be approximated by a Rayleigh distribution for a narrowband process,

$$P(S) = \frac{S}{\sigma_s^2} e^{-\frac{S^2}{2\sigma_s^2}} \quad , \quad S \geq 0 \quad , \quad (9)$$

where σ_s^2 is the mean square stress level.

Approximate Methods for Single Degree of Freedom Systems

For a linear single degree of freedom system, Miles³ has replaced $n(S)$ in equation (7) with the system resonant frequency in Hertz, $\omega_0/2\pi$. Equations (7) and (9) then give Miles' single degree of freedom formula³,

$$E[D_m] = \frac{\omega_0}{2\pi c} (\sqrt{2} \sigma_s)^b \Gamma\left(\frac{b+2}{2}\right) \quad , \quad (10)$$

where,

$$\Gamma(y) = 2 \int_0^{\infty} x^{2y-1} e^{-x^2} dx \quad , \quad y > 0 \quad . \quad (11)$$

If a single degree of freedom oscillator is nonlinear it is possible to evaluate equation (7) by replacing $n(S)$ with the expected total number of peaks per unit time, $E[M_T]$. If we again assume that the response process is narrowband, it is not difficult

to approximate the peak probability density, $P(S)$. If the displacement response, $x(t)$ is governed by Duffing's equation,

$$\ddot{x} + \omega_0^2 (x + \epsilon x^3) + \alpha \dot{x} = f(t) \quad , \quad (12)$$

where ω_0 is the linear natural frequency, ϵ and α are constants, and $f(t)$ is Gaussian white noise, then the damage rate may be approximated by⁴,

$$E[D_m] = \frac{1}{C} C \omega_0^2 K^b \int_0^{\infty} x^b (x + \epsilon x^3) e^{-\frac{x^2}{2\sigma_0^2} - \frac{\epsilon x^4}{4\sigma_0^2}} dx \quad , \quad (13)$$

where

$$\frac{1}{C} = \sqrt{2\pi\omega_0^2\sigma_0^2} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma_0^2} - \frac{\epsilon x^4}{4\sigma_0^2}} dx \quad , \quad (14)$$

and σ_0^2 is the mean square displacement response when $\epsilon=0$, and $\sigma_s^2 = G_f / (4\omega_0^2\alpha)$, where G_f is the constant single-sided power spectral density of the excitation in units of f^2/Hertz . The constant K in equation (13) is an assumed (for the present study) linear relation between stress³ and displacement, i.e.

$$S = K x \quad . \quad (15)$$

When $\epsilon=0$ equation (13) reduces to equation (10).

An alternative approach to estimate the fatigue life is to solve equation (12) numerically and use equation (15) to determine the stress as a function of time. When a stress reversal occurs at the stress level S , one simply increments the accumulated damage according to

$$D_m = D_m + \frac{|S|^b}{C} \quad , \quad (16)$$

where D_m is initially set to zero. If the solution is simulated for a time period equal to τ then the damage rate is D_m/τ and the predicted fatigue life is

$$T = \frac{\tau}{D_m} \quad . \quad (17)$$

While the above methods are straight-forward to use for simple systems with a single degree of freedom such as those governed by equation (12), there are substantial difficulties in applying them to more complex multi-degree of freedom structures. The derivations of equations (10) or (13) rely on the assumption that the response process has a very narrow bandwidth, which is expected to be true only for a lightly damped single degree of freedom oscillator. Numerical methods such as equation (16) are computationally impractical when simulating the random response of a complex nonlinear system. Since most structures of practical interest are too complicated to analyze using existing techniques there is a need to develop more powerful procedures.

Fatigue Life Estimates Using Equivalent Linearization

In this section, methods for estimating fatigue life based on conventional equivalent linearization⁵ and by a modified approach will be described. The modified equivalent linearization method discussed here follows from reference [1] where it is shown that this technique gives very accurate estimates of the power spectral density of the response of a nonlinear system which is a dramatic improvement over the conventional approach. It will be shown that both of these theories may be adapted to give reasonable fatigue life estimates for a system with nonlinear stiffness that has a single resonant mode.

The method of equivalent linearization consists of approximating equation (12) by

$$\ddot{x} + \omega_e^2 x + \alpha \dot{x} + e(x) = f(t) \quad , \quad (18)$$

where ω_e is the equivalent linear natural frequency and $e(x)$ is the error in the approximation. ω_e is chosen so that $e(x)$ is minimized. The main difference between the conventional approach and the modified method¹ is that in the modified method,

ω_e is taken to be random rather than deterministic. One may then avoid the assumption that the response is a Gaussian random process.

In conventional equivalent linearization⁵ the square of the equivalent linear natural frequency is given by

$$\omega_e^2 = \frac{\omega_o^2}{2} \left(1 + \sqrt{1 + 12\varepsilon\sigma_o^2} \right) \quad , \quad (19)$$

where, as before

$$\sigma_o^2 = \frac{G_f}{4\omega_o^2\alpha} \quad (20)$$

is the mean square response when $\varepsilon=0$.

The mean square value of the response of the nonlinear system is then approximated by

$$\sigma_x^2 = \frac{G_f}{4\omega_e^2\alpha} \quad (21)$$

To obtain an estimate of the fatigue life of a system governed by equation (12) we could simply calculate the fatigue life of an equivalent linear system such as in equation (18) with $e(x)$ neglected. This may be accomplished by using equation (10) with ω_o replaced by ω_e and σ_s replaced by $K\sigma_x$,

$$E[D_m] = \frac{\omega_e}{2\pi C} \left(\sqrt{2} K \sigma_x \right)^b \Gamma\left(\frac{b+2}{2}\right) \quad (22)$$

This result is derived with the assumption that the peak probability function is a Rayleigh distribution which corresponds to a narrow-band Gaussian random process. This assumption may be avoided when the modified method of reference [1] is used. In reference [1] the system is considered to be excited by a driver having a randomly varying power spectral density. Since the excitation is Gaussian, the input spectrum level may be described in terms of a chi-square random variable, ξ .

The random fluctuations in ω_e^2 are then described in terms of ξ , where ξ is exponentially distributed, $P(\xi) = e^{-\xi}$, $\xi \geq 0$, and

$$\omega_e^2(\xi) = \frac{\omega_o^2}{2} \left(1 + \sqrt{1 + 12\epsilon k \sigma_o^2 \xi} \right) \quad (23)$$

where k is obtained by iteratively solving

$$k = \frac{\int_0^{\infty} \left(\frac{\xi}{1 + \sqrt{1 + 12\epsilon k \sigma_o^2 \xi}} \right)^2 e^{-\xi} d\xi}{3 \left(\int_0^{\infty} \frac{\xi e^{-\xi} d\xi}{1 + \sqrt{1 + 12\epsilon k \sigma_o^2 \xi}} \right)^2} \quad (24)$$

In reference [1] expected values are calculated in terms of integrations over $P(\xi)$ rather than in terms of the response variable, x , as in the conventional formulation. Since the probability density of x is usually unknown, a Gaussian distribution is typically assumed. Equation (7) may be transformed into an integration over ξ as

$$E[D_m] = \int_0^{\infty} n(\xi) \frac{|S(\xi)|^p}{c} P(\xi) d\xi \quad (25)$$

From equation (19) we may set

$$n(\xi) = \frac{\sqrt{\omega_e^2(\xi)}}{2\pi} = \frac{\omega_o}{2\pi\sqrt{2}} \sqrt{1 + \sqrt{1 + 12\epsilon k \sigma_o^2 \xi}} \quad (26)$$

In reference [1] a result is presented for the 'mean-square' response, $\sigma_x^2(\xi)$, corresponding to each value of ξ ,

$$\sigma_x^2(\xi) = \frac{2 \xi \sigma_o^2}{1 + \sqrt{1 + 12\epsilon k \sigma_o^2 \xi}} \quad (27)$$

In equation (25) we require an expression for the peak stress as a function of ξ . An approximation for the peak response level, $|X_p(\xi)|$ may be obtained from equation (27) by considering the response corresponding to a particular value of ξ to be

dominated by a single frequency component at the frequency $\omega_e(\xi)$. With this assumption the relation between the mean square and the peak response is

$$\sigma_x^2(\xi) = \frac{|x_p(\xi)|^2}{2} \quad , \quad \text{or} \quad |x_p(\xi)| = \sqrt{2 \sigma_x^2(\xi)} \quad (28)$$

From equation (15) and (28) the peak stress may be written as

$$|S(\xi)| = K \sqrt{2 \sigma_x^2(\xi)} \quad (29)$$

Equations (25) through (29) then give

$$E[D_m] = \frac{2^{b-\frac{1}{2}} \omega_o K^b \sigma_o^b}{2\pi} \int_0^\infty \left(1 + \sqrt{1 + 12\epsilon k \sigma_o^2 \xi}\right)^{\frac{1-b}{2}} \xi^{\frac{b}{2}} e^{-\xi} d\xi \quad (30)$$

The fatigue life is obtained from equations (8) and (30). When $\epsilon=0$, equation (30) reduces to equation (10) for the linear system.

Governing Equation for a Nonlinear Plate

To compare estimates of the fatigue life of a system using the methods of the previous sections we will calculate the coefficients of Duffing's equation (12) corresponding to the response of a nonlinear plate with simply-supported boundaries. We will assume that the plate response may be described by a single resonant mode. Since we are only attempting to compare solution methods it is sufficient to use an approximation to obtain the coefficients of equation (12). It is important, however, to perform the comparisons in a region of parameter space that is physically relevant.

If the in-plane motion of the plate is constrained at the edges, the Berger hypothesis⁶ leads to

$$D\nabla^4 w - \rho h e C_p^2 (w_{,xx} + w_{,yy}) + \rho h \ddot{w} + \alpha \dot{w} = p(x, y, t) \quad (31)$$

where

$$e = \frac{1}{2L_x L_y} \int_0^{L_y} \int_0^{L_x} (w_{,x}^2 + w_{,y}^2) dx dy \quad (32)$$

w is the transverse displacement, ρ is the density, h is the thickness, α is a viscous damping coefficient, and $p(x,y,t)$ is the applied pressure. L_x and L_y are the dimensions of the plate in the x and y directions. The constants D and C_p are the bending rigidity and wave speed,

$$D = \frac{E h^3}{12(1-\nu^2)} \quad , \quad C_p^2 = \frac{E}{\rho(1-\nu^2)} \quad (33)$$

E is Young's modulus and ν is Poisson's ratio. Comma denotes partial differentiation. For a simply supported plate,

$$w(x,y,t) = A(t) \sin\left(\frac{\pi x}{L_x}\right) \sin\left(\frac{\pi y}{L_y}\right) \quad , \quad (34)$$

where $A(t)$ is an unknown function of time.

By substituting equation (34) into equations (31) and (32) the governing equation takes the form of equation (12),

$$\ddot{A} + \omega_0^2 (A + \varepsilon A^3) + \alpha \dot{A} = F(t) \quad , \quad (35)$$

where,

$$\omega_0^2 = \frac{D}{\rho h} \left[\left(\frac{\pi}{L_x} \right)^2 + \left(\frac{\pi}{L_y} \right)^2 \right]^2 \quad , \quad (36)$$

$$\varepsilon = \frac{3}{2h^2} \quad , \quad (37)$$

and

$$F(t) = \frac{4}{\rho h L_x L_y} \int_0^{L_y} \int_0^{L_x} \sin\left(\frac{\pi x}{L_x}\right) \sin\left(\frac{\pi y}{L_y}\right) p(x,y,t) dx dy \quad . \quad (38)$$

Let the excitation have the form

$$p(x,y,t) = p_0(t) \sin\left(\frac{\pi x}{L_x}\right) \sin\left(\frac{\pi y}{L_y}\right) \quad . \quad (39)$$

Equations (38) and (39) give

$$F(t) = \frac{p_o(t)}{\rho h} \quad (40)$$

The damping coefficient, α , in equation (35) is taken to be

$$\alpha = \omega_o \eta \quad (41)$$

where η is the loss factor.

For the present study we will obtain the stress-displacement relation, equation (15), by neglecting in-plane displacements and assuming that the maximum stress may be approximated by

$$S = E \epsilon_y \quad (42)$$

where ϵ_y is the strain in the y direction at the surface of the plate at its mid-point ($x = L_x/2, y = L_y/2$). For a simply-supported plate this will be the location of maximum stress. It is assumed that the y component of the strain is greater than the x component which will be true if $L_x > L_y$. Evaluating the stress in this case gives

$$S = -E \frac{h}{2} \frac{\partial^2 w}{\partial y^2} = E \frac{h}{2} \left(\frac{\pi}{L_y} \right)^2 A \sin^2 \frac{\pi}{2} = \frac{Eh}{2} \left(\frac{\pi}{L_y} \right)^2 A \quad (43)$$

The constant K in equation (15) is then

$$K = \frac{Eh}{4} \left(\frac{\pi}{L_y} \right)^2 \quad (44)$$

This is, of course, a very crude approximation but it will suffice for our present purposes.

The remaining constants to be determined for our comparisons are the material properties b and c in the damage accumulation model, equation (6). These must be obtained experimentally. For this study, we only need to use 'typical' numbers for a common material. The constants used in the calculations are

$$b = 6.33 \text{ and } c = 6.56 \times 10^{30} \quad (45)$$

The results presented in the following correspond to an aluminum plate with the following properties: (English units)

$$E = 10^7 \text{ PSI}, \rho = 0.1 \text{ lbs./in}^3, \nu = 0.3, L_x = 20 \text{ in.},$$

$$L_y = 10 \text{ in.}, h = 0.063 \text{ in.}, \text{ and } \eta = 0.05.$$

The excitation is assumed to result from acoustic pressure fluctuations. The driving sound pressure level is given by

$$L_p = 20 \log \frac{p}{p_{\text{ref}}} \quad (46)$$

where $p_{\text{ref}} = 0.0002 \mu \text{ bar}$. A pressure of one pound per square inch corresponds to a sound level of approximately 170 dB.

Results and Recommendations

The calculated results are given in figure 2 for a range of excitation levels from 130 to 155 dB. The predicted fatigue lives based on damage rates estimated using the 'classical' result, equation (13), numerical simulation as in equation (16), and the methods based on equivalent linearization, equations (22) and (30), are found to be in remarkably close agreement. For comparison, the fatigue life obtained for a linear system ($\epsilon=0$) is also shown. Since the nonlinearity limits the response amplitude in this case, the linear system has a shorter fatigue life. This may not be true in general, however, since in the present study a linear stress displacement relation (equation (15)) has been assumed. The effects of nonlinear in-plane strains should be included in future studies.

The accuracy of equivalent linearization methods in predicting the fatigue life when the response deviates significantly from the linear behavior as shown in figure 2 is quite remarkable. The method of equivalent linearization provides the parameters of a linear system which has 'nearly' the same mean-square response as the nonlinear system being studied. However, higher order statistics of the output of the equivalent linear system which are thought to strongly influence fatigue life (such as distributions of peaks, level crossing and spectra) differ substantially from

those of the nonlinear system. In this case, as often occurs in nonlinear analyses, the range of validity of the approximate solution methods are far from obvious and can be quite difficult to determine. The numerical results presented here indicate that the fatigue life of a nonlinear system with a single degree of freedom depends primarily on the mean square response level and the average frequency. A more detailed analysis is needed to verify this result.

An examination of the error in the fatigue life as predicted using equivalent linearization could proceed by comparing the equations for the expected value of the damage rate, equations (19) through (22) with the probabilistic formulation in equations (13) and (14). It should be possible to obtain an expression for the difference between equations (13) and (22) which would explicitly show how the parameters of the system influence of the error. This analysis might lead to an improved approximate formulation in which the fatigue life is estimated directly rather than needing to first linearize the system. An approach of this type would be beneficial since it would be easier to see how the fatigue life is influenced by the system properties.

While a goal of this investigation has been to improve our understanding of the fatigue of complex aerospace structures, the present study has been limited to systems which have a single degree of freedom. However, measurements of the response of stiffened fuselage structures can exhibit a nearly uncountable number of resonant modes. This is illustrated in figure 3 which shows the power spectral density of the strain at the location of maximum strain in a realistic test structure.⁷ The structure consists of a panel with orthogonal stiffeners such that there are a total of nine skin panels. The figure shows a large number of spikes which presumably correspond to the response of resonant modes. In acoustic fatigue studies it is common practice to treat structures such as that in figure 3 as a single degree of

freedom system. In reference [7] this structure was considered to have only one resonant mode with a frequency of 380 Hz. It may be argued that this approach works because it has lead to successful designs. It is, however, far from clear (at least to the author) why this works. Considering the structure to be a single degree of freedom system, which it obviously is not, is not at all appealing and could yield incorrect designs. In order to develop reliable acoustic fatigue design procedures for future materials and structural configurations there is a need to extend the existing analysis methods to include many degrees of freedom. From the results presented here, equivalent linearization appears to be a promising method which could be adapted for fatigue life predictions of nonlinear structures with multiple modes. The present numerical study and the suggested analysis discussed above for the single mode case should be repeated with multiple modes included.

Conclusions

The results presented here show that equivalent linearization produces estimates of the fatigue life of a nonlinear plate which agree closely with those produced by other methods that describe the statistics of the response much more accurately. The present analysis is limited, however, to single degree of freedom systems that may be described by Duffing's equation. Further analytical work is needed to examine the applicability of equivalent linearization in fatigue life prediction and in the development of approximate methods for predicting the fatigue of complex nonlinear structures with multiple degrees of freedom.

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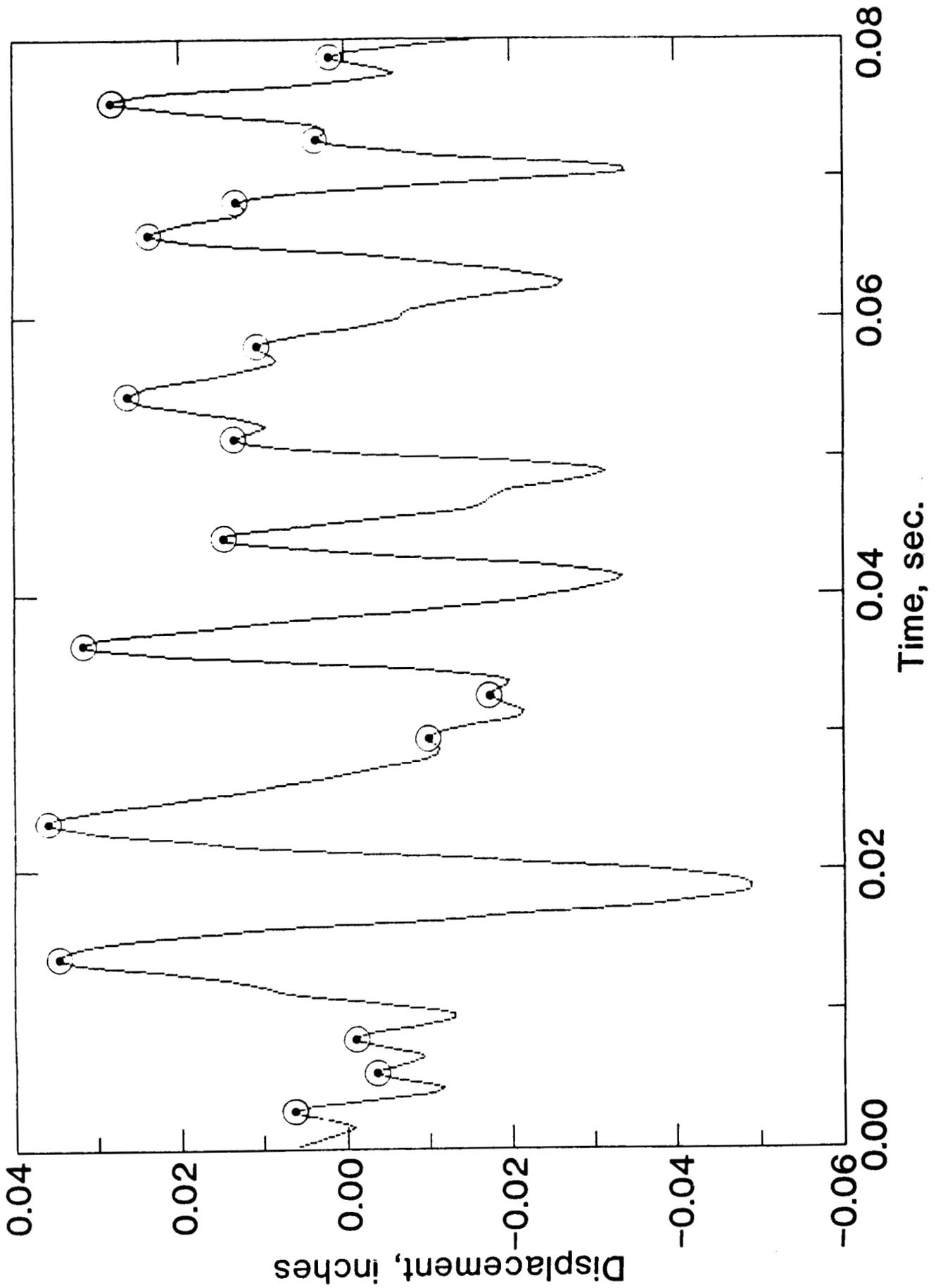
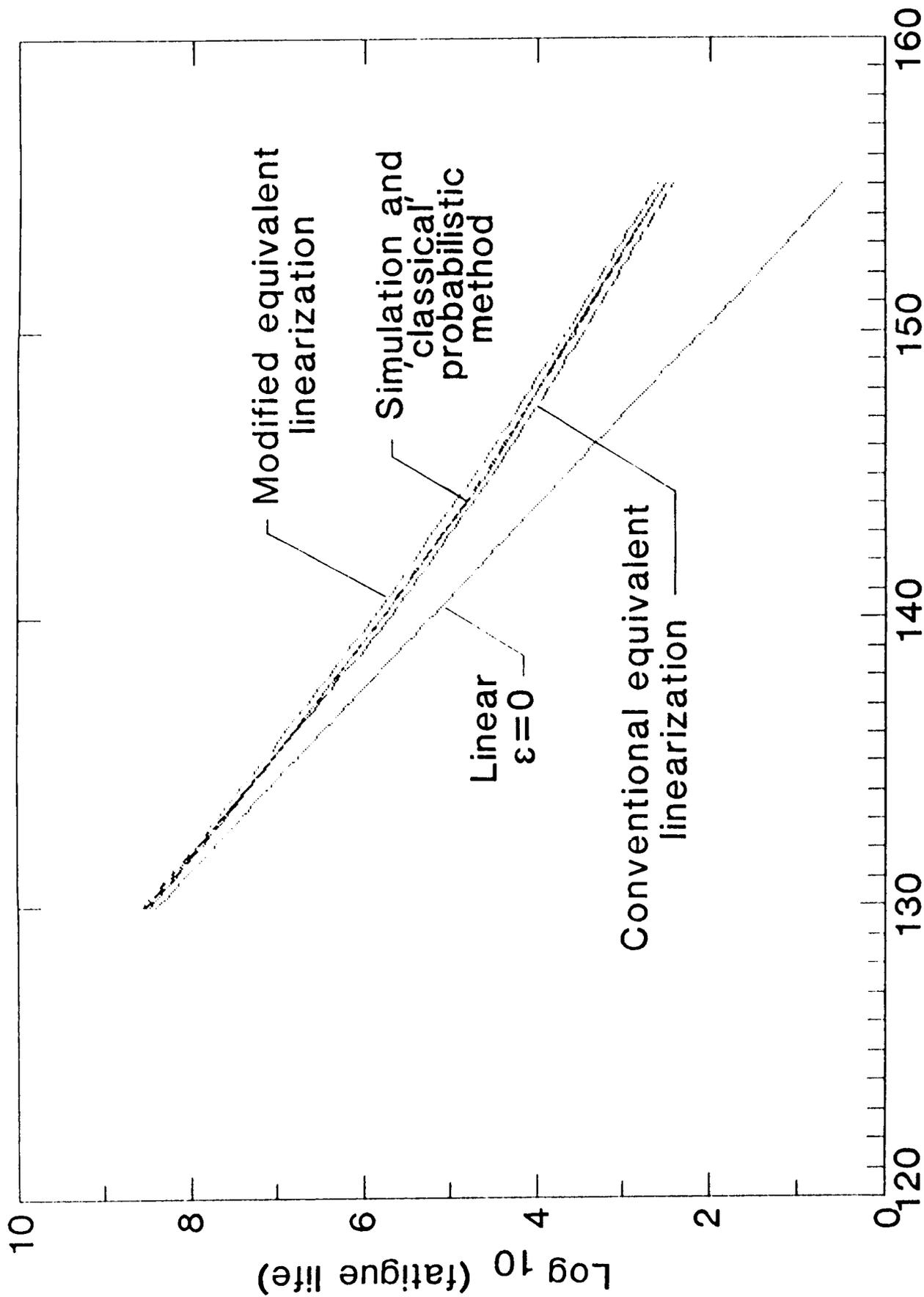


Figure 1. Simulated response time history.



Excitation sound level, dB

Figure 2. Predicted fatigue life of a nonlinear plate.

Panel configuration: s
Transducer: G10
Overall R.M.S. level: $261.8 \mu\epsilon$
Input spectrum: random
Input level: 155 dB

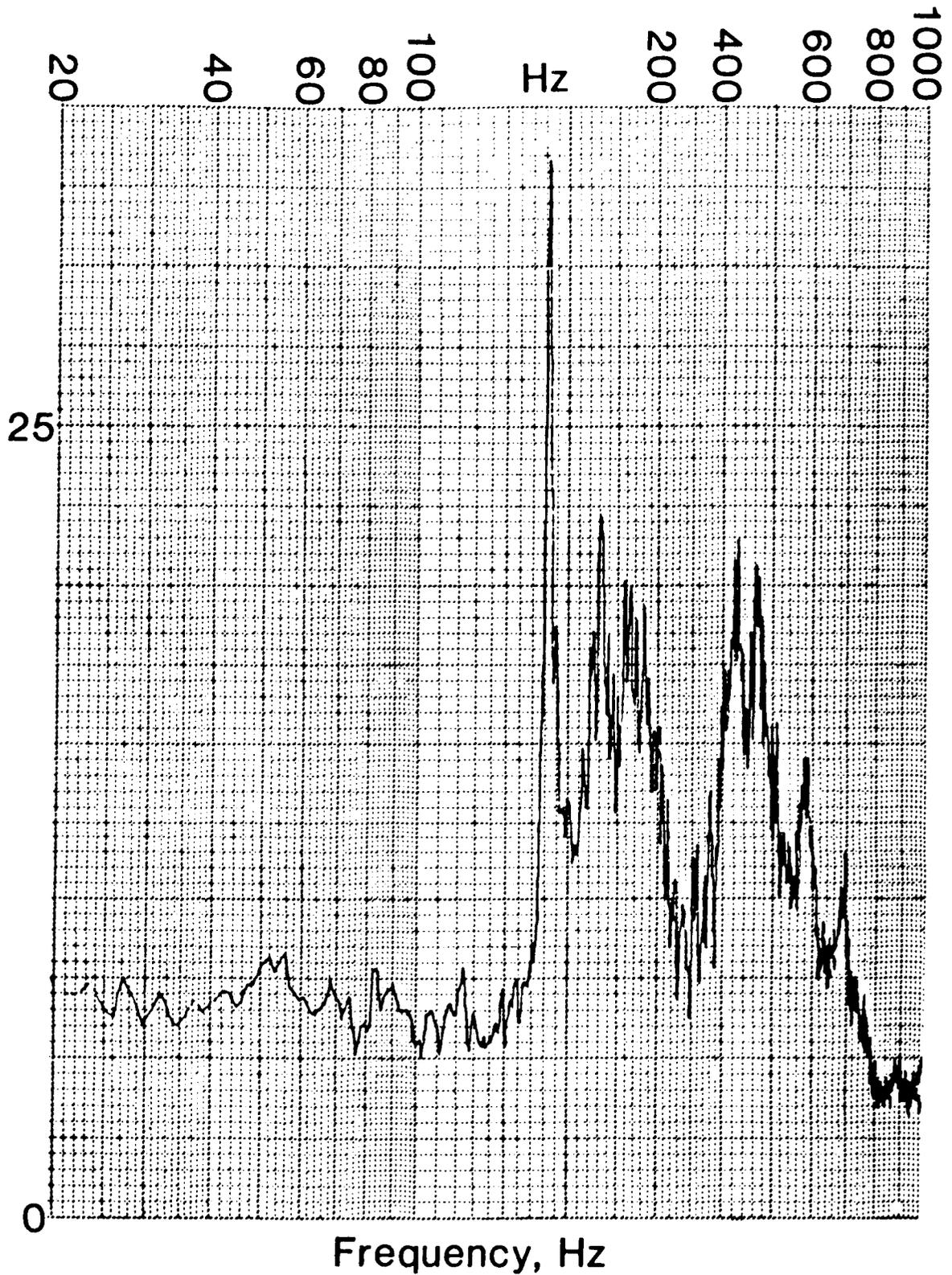


Figure 3. Strain spectrum for panel s.
(From Reference (7)).



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